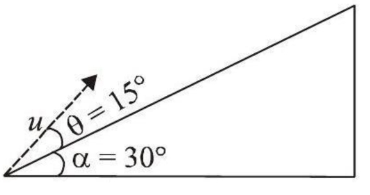
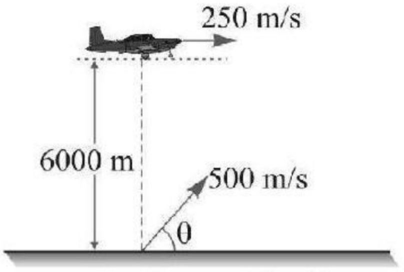
Motion in a Plane

1. A body is projected at $t = 0$ with a velocity $10\ \mathrm{ms^{-1}}$ at an angle of 60° with the horizontal. The radius of curvature of its trajectory at $\mathrm{t} = 1\ \mathrm{s}$ is Rm. Neglecting air resistance and taking acceleration due to gravity $g = 10\ \mathrm{ms^{-2}}$, the value of R is:
2. A particle is moving along a circular path with a constant speed of $10\ \mathrm{ms^{-1}}$. What is the magnitude of the change in velocity (in $\mathrm{m/s}$) of the particle, when it moves through an angle of 60° around the centre of the circle?
3. A particle moves from the point $(2.0\hat{\imath} + 4.0\hat{\jmath})\mathrm{m}$, at $\mathrm{t} = 0$, with an initial velocity $(5.0\hat{\imath} + 4.0\hat{\jmath})\mathrm{ms^{-1}}$. It is acted upon by a constant force which produces a constant acceleration $(4.0\hat{\imath} + 4.0\hat{\jmath})\mathrm{ms^{-2}}$. What is the distance (in metre) of the particle from the origin at time 2 s ?
4. Ship A is sailing towards north-east with velocity $\vec{v} = 30\hat{\imath} + 50\hat{\jmath}\ \mathrm{km/hr}$ where $\hat{\imath}$ points east and $\hat{\jmath}$, north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at $10\ \mathrm{km/hr}$. A will be at minimum distance from B in hours.
5. The position vector of a particle changes with time according to the relation $\vec{r}(\mathrm{t}) = 15\mathrm{t}^2\hat{\imath} + (4 - 20\mathrm{t}^2)\hat{\jmath}$. What is the magnitude of the acceleration (in $\mathrm{m/s^2}$) at $t = 1$?
6. A plane is inclined at an angle $\alpha = 30^\circ$ with respect to the horizontal. A particle is projected with a speed $\mathrm{u} = 2\ \mathrm{ms^{-1}}$, from the base of the plane, as shown in figure. The distance (in metre) from the base, at which the particle hits the plane is close to : (Take $\mathrm{g} = 10\ \mathrm{ms^{-2}}$)



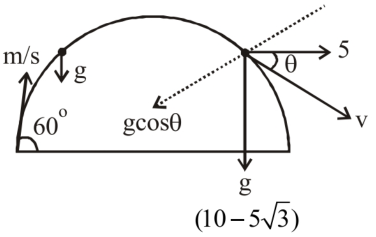
7. The angle between the two vectors $\vec{A} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ and $\vec{B} = 3\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ will be
8. A vector is represented by $3\hat{\imath} + \hat{\jmath} + 2\hat{k}$. Projection of this vector in XY plane is
9. The resultant of two vectors $\vec{A}$ and $\vec{B}$ is perpendicular to the vector $\vec{A}$ and its magnitude is equal to half the magnitude of vector $\vec{B}$. The angle between $\vec{A}$ and $\vec{B}$ is
10. A person aiming to reach the exactly opposite point on the bank of a stream is swimming with a speed of $0.5\ \mathrm{m/s}$ at an angle of 120° with the direction of flow of water. The speed (in $\mathrm{m/s}$) of water in the stream is
11. The horizontal range of a projectile is $4\sqrt{3}$ times its maximum height. Its angle of projection will be
12. An aircraft moving with a speed of $250\ \mathrm{m/s}$ is at a height of $6000\ \mathrm{m}$, just overhead of an anti-aircraft gun. If the muzzle velocity is $500\ \mathrm{m/s}$, the firing angle θ should be:



13. A body is thrown horizontally from the top of a tower of height 5 m. It touches the ground at a distance of 10 m from the foot of the tower. The initial velocity (in $\mathrm{m/s}$) of the body is ($g = 10\ \mathrm{ms}^{-2}$)
14. At the height 80 m, an aeroplane is moving with a speed of $150\ \mathrm{m/s}$. A bomb is dropped from it so as to hit a target. At what distance (in metre) from the target should the bomb be dropped (given $g = 10\ \mathrm{m/s^2}$)
15. Two bodies are thrown up at angles of $45°$ and $60°$ respectively, with the horizontal. If both bodies attain same vertical height, then the ratio of velocities with which these are thrown is $\sqrt{\frac{x}{2}}$. Find the value of x.

SOLUTIONS

1. **(2.8)**



Horizontal component of velocity

$v_x = 10\cos 60° = 5$ m/s

vertical component of velocity

$v_y = 10\cos 30° = 5\sqrt{3}$ m/s

After t = 1 sec.

Horizontal component of velocity $v_x = 5$ m/s

Vertical component of velocity

$v_y = |(5\sqrt{3} - 10)|\ m/s = 10 - 5\sqrt{3}$

Centripetal, acceleration $a_n = \frac{v^2}{R}$

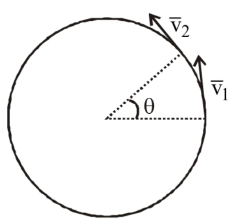
$\Rightarrow R = \frac{v_x^2 + v_y^2}{a_n} = \frac{25 + 100 + 75 - 100\sqrt{3}}{10\cos\theta}$ ...(i)

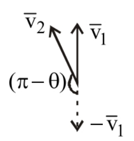
From figure (using (i))

$\tan\theta = \frac{10 - 5\sqrt{3}}{5} = 2 - \sqrt{3} \Rightarrow \theta = 15°$

$R = \frac{100(2 - \sqrt{3})}{10\cos 15} = 2.8\,m$

2. **(10)**





Change in velocity,

$|\Delta\bar{v}| = \sqrt{v_1^2 + v_2^2 + 2v_1 v_2 \cos(\pi - \theta)}$

$= 2v\sin\frac{\theta}{2}$ $\quad (\because |\vec{v}_1| = |\vec{v}_2|) = v$

$= (2 \times 10) \times \sin(30°) = 2 \times 10 \times \frac{1}{2}$

$= 10$ m/s

3. **(28.28)** As $\vec{S} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$$\vec{S} = (5\hat{i} + 4\hat{j})2 + \frac{1}{2}(4\hat{i} + 4\hat{j})4$$

$$= 10\hat{i} + 8\hat{j} + 8\hat{i} + 8\hat{j}$$

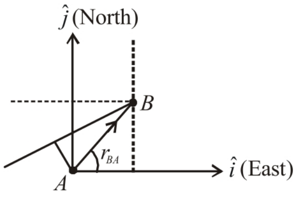
$$\vec{r}_f - \vec{r}_i = 18\hat{i} + 16\hat{j}$$

[as $\vec{s}$ = change in position = $\vec{r}_f - \vec{r}_i$]

$$\vec{r}_r = 20\hat{i} + 20\hat{j}$$

$$|\vec{r}_r| = 20\sqrt{2}$$

4. **(2.6)**



$$\vec{v}_A = 30\hat{i} + 50\hat{j} \text{ km/hr}$$

$$\vec{v}_B = (-10\hat{i}) \text{ km/hr}$$

$$r_{BA} = (80\hat{i} + 150\hat{j}) \text{ km}$$

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = -10\hat{i} - 30\hat{i} - 50\hat{i}$$

$$= 40\hat{i} - 50\hat{j}$$

$$t_{\text{minimum}} = \frac{|(\vec{r}_{BA})\cdot(\vec{v}_{BA})|}{|(\vec{v}_{BA})|^2}$$

$$= \frac{|(80\hat{i} + 150\hat{j})(-40\hat{i} - 50\hat{j})|}{(10\sqrt{41})^2}$$

$$\therefore\ t = \frac{10700}{10\sqrt{41} \times 10\sqrt{41}} = \frac{107}{41} = 2.6 \text{ hrs.}$$

5. **(25)** $\vec{r} = 15t^2\hat{i} + (4 - 20t^2)\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = 30t\hat{i} - 40t\hat{j}$$

Acceleration, $\vec{a} = \frac{d\vec{v}}{dt} = 30\hat{i} - 40\hat{j}$

$$\therefore a = \sqrt{30^2 + 40^2} = 50 \text{ m/s}^2$$

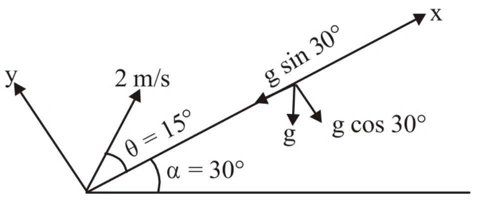
6. **(0.195)** On an inclined plane, time of flight (T) is given by

$$T = \frac{2u\sin\theta}{g\cos\alpha}$$

Substituting the values, we get

$$T = \frac{(2)(2\sin 15°)}{g\cos 30°} = \frac{4\sin 15°}{10\cos 30°}$$

Distance, S $= (2\cos 15°)T - \frac{1}{2}g\sin 30°(T)^2$



$$= (2\cos 15°)\frac{4}{10}\frac{\sin 15°}{10\cos 30°} - \left(\frac{1}{2}\times 10\sin 30°\right)\frac{16\sin^2 15°}{100\cos^2 30°}$$

$$= \frac{16\sqrt{3}-16}{60} \simeq 0.195\text{m}$$

7. **(90°)** $\vec{A} = 3\hat{i}+4\hat{j}+5\hat{k}$, $\vec{B} = 3\hat{i}+4\hat{j}-5\hat{k}$

$\vec{A}\,.\,\vec{B} = (3\hat{i}+4\hat{j}+5\hat{k})\,.\,(3\hat{i}+4\hat{j}-5\hat{k})$

$|\vec{A}|\,|\vec{B}|\cos\theta = 9+16-25 = 0$

$|\vec{A}| \neq 0$, $|\vec{B}| \neq 0$, hence, $\cos\theta = 0 \Rightarrow \theta = 90°$

8. **(3.16)** $\vec{R} = 3\hat{i}+\hat{j}+2\hat{k}$

$\therefore$ Length in XY plane $= \sqrt{R_x^2+R_y^2} = \sqrt{3^2+1^2} = \sqrt{10}$

9. **(150°)** $\frac{B}{2} = \sqrt{A^2+B^2+2AB\cos\theta}$ ...... (i)

$\therefore \tan 90° = \frac{B\sin\theta}{A+B\cos\theta} \Rightarrow A + B\cos\theta = 0$

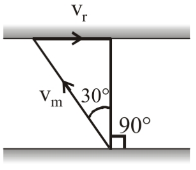
$\therefore \cos\theta = -\frac{A}{B}$

Hence, from (i) $\frac{B^2}{4} = A^2+B^2-2A^2 \Rightarrow A = \sqrt{3}\frac{B}{2}$

$\Rightarrow \cos\theta = -\frac{A}{B} = -\frac{\sqrt{3}}{2} \quad \therefore \theta = 150°$

10. **(0.25)** $\sin 30° = \frac{v_r}{v_m} = \frac{1}{2}$

$\Rightarrow v_r = \frac{v_m}{2} = \frac{0.5}{2} = 0.25$ m/s.



11. **(30°)** $R = 4H\cot\theta$, if $R = 4\sqrt{3}H$ then

$\cot\theta = \sqrt{3} \Rightarrow \theta = 30°$

12. **(60°)** $500\cos\theta = 250 \Rightarrow \cos\theta = \frac{1}{2}$

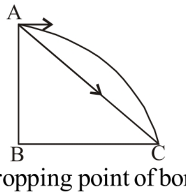
or $\theta = 60°$.

13. **(10)** $S = u\times\sqrt{\frac{2h}{g}} \Rightarrow 10 = u\sqrt{2\times\frac{5}{10}} \Rightarrow u = 10\text{ m/s}$

14. **(605.3)** The horizontal distance covered by bomb,

$BC = u_H \times \sqrt{\frac{2h}{g}}$

$= 150\sqrt{\frac{2\times 80}{10}} = 600\text{m}$



$\therefore$ The distance of target from dropping point of bomb.

$AC = \sqrt{AB^2+BC^2} = \sqrt{(80)^2+(600)^2} = 605.3\text{m}$

15. **(3)** $H_{max} = \frac{u^2\sin^2 q}{2g}$

According to question, $\frac{u_1^2\sin^2 45°}{2g} = \frac{u_2^2\sin^2 60°}{2g}$

$\Rightarrow \frac{u_1^2}{u_2^2} = \frac{\sin^2 60°}{\sin^2 45°} \Rightarrow \frac{u_1}{u_2} = \frac{\sqrt{3}/2}{1/\sqrt{2}} = \sqrt{\frac{3}{2}}.$